

Dilaton Dark Energy Model in $f(R)$, $f(T)$ and Hořava-Lifshitz Gravities

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In this work, we have considered dilaton dark energy model in Weyl-scaled induced gravitational theory in presence of barotropic fluid. It is to be noted that the dilaton field behaves as a quintessence. Here we have discussed the role of dilaton dark energy in modified gravity theories namely, $f(R)$, $f(T)$ and Hořava-Lifshitz gravities and analyzed the behaviour of the dilaton field and the corresponding potential in respect to these modified gravity theories instead of Einstein's gravity. In $f(R)$ and $f(T)$ gravities, we have considered some particular forms of $f(R)$ and $f(T)$ and we have shown that the potentials always increase with the dilaton fields. But in Hořava-Lifshitz gravity, it has been seen that the potential always decreases as dilation field increases.

I. INTRODUCTION

Recent observations indicate that our universe is currently expanding with an acceleration [1]. The main responsible candidate for this acceleration is dark energy (DE). There are several types of exotic type of dark energy with negative pressure depending on their equation of state (EOS). When $-1 < w < -1/3$, it is called quintessence and when $w < -1$, it is phantom. There are some other dark energies model which can cross the phantom divide $w = -1$ both sides are called quintom. Recently such acceleration is understood by imposing a concept of modification of gravity for an alternative candidate of dark energy [2 - 40]. This concept provides very natural gravitational alternative for exotic matter. This type of gravity is predicted by string/M-theory. The explanation of the phantom or non-phantom or quintom phase of the universe can be described by this gravity without introducing negative kinetic term of dark energies. After proposing dark energy models to explain cosmic acceleration it was realized that even non linear terms of Ricci curvature R^{-n} ($n > 0$) can be used as an alternative of DE [2 - 15]. Several authors considered different form of $f(R)$ which exhibit late time acceleration for small curvature and inflation for large curvature. The modified gravity theory can be constructed by adding geometrical correction terms to the usual Einstein-Hilbert Lagrangian L considering as a function of scalar curvature $L = f(R)$. $f(R)$ model has the importance that it satisfy both the cosmological as well as local gravity constraints.

Another approach can be explored using Weitzenböck connection having no curvature but torsion which is formed from products of first derivatives of the tetrad with no second derivatives in the torsion tensor. It is extremely relate to general relativity only exception in boundary terms [16 - 22]. In this modified gravity approach $f(T)$ torsion will be responsible candidate of the observed acceleration of the universe without restoring the DE. An advantage of the generalized $f(T)$ theory than $f(R)$ theory is its field equations of second order compare to forth ordered equations of $f(R)$ theory.

Recently Hořava [23] proposed a new theory of gravity. It is renormalizable with higher spatial derivatives in four dimensions which reduces to Einstein's gravity with non vanishing cosmological constant in IR but with improved UV behaviours. It is similar to a scalar field theory of Lifshitz [24] in which the time dimension has weight 3 if a space dimension has weight 1 and this theory is called Hořava-Lifshitz gravity. Hořava-Lifshitz gravity has been studied and extended in detail and applied as a cosmological framework of the universe [25 - 40].

One can construct a dilaton dark energy model as non-minimal quintessence based on Weyl-scaled induced gravitational theory to find an important thing that when dilaton field is not gravitational clustered at small

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scales, the effect of dilation can not change the evolutionary law of structure formation [41, 42]. Quintessence dark energy models have been widely studied and here we consider dilaton dark energy model in Weyl-scaled induced gravitational theory as a quintessence [41 - 56] in $f(R)$, $f(T)$ and Hořava-Lifshitz gravities and analyze the behaviour of the potential and field in respect to these modified dark energy models.

II. BASIC EQUATIONS IN DILATON DARK ENERGY MODEL

The action of the Weyl-scaled induced gravitational theory is given by [42,45]

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R(g_{\mu\nu}) - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V(\sigma) + \frac{1}{2} g^{\mu\nu} e^{-\alpha\sigma} \partial_\mu \phi \partial_\nu \phi - e^{-2\alpha\sigma} V(\phi) \right] \quad (1)$$

where $\alpha = \sqrt{\frac{\kappa^2}{2\varpi+3}}$ with ϖ being an important parameter in Weyl-scaled induced gravitational theory. Here σ is the dilation field and $V(\sigma)$ is the scalar potential of the dilation field.

By varying action (1) and considering dilaton field as the candidate of DE, the field equations and the conservation equations in Friedmann-Robertson-Walker model, become ($\kappa^2 \equiv 8\pi G$):

$$H^2 + \frac{k}{a^2} = \frac{\kappa^2}{3}(\rho_m + \rho_\sigma) \quad (2)$$

$$\dot{H} - \frac{k}{a^2} = -\frac{\kappa^2}{2}(\rho_m + p_m + \rho_\sigma + p_\sigma) \quad (3)$$

$$\dot{\rho}_m + 3H(\rho_m + p_m) = \frac{1}{2}\alpha\dot{\sigma}(\rho_m + 3p_m) \quad (4)$$

and

$$\dot{\rho}_\sigma + 3H\dot{\sigma}^2 = \frac{1}{2}\alpha e^{-\alpha\sigma}(\rho_m - 3p_m) \quad (5)$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, ρ_m is dark matter energy density, ρ_σ is dilaton dark energy density and radiation is neglected. The effective energy density and pressure of dilaton dark energy are given by,

$$\rho_\sigma = \frac{1}{2}\dot{\sigma}^2 + V(\sigma) \quad (6)$$

and

$$p_\sigma = \frac{1}{2}\dot{\sigma}^2 - V(\sigma) \quad (7)$$

For matter, $p_m = 0$ we have from equation (4) $\rho_m = \rho_{m0}a^{-3}e^{\frac{\alpha\sigma}{2}}$ as the matter density. For perfect fluid with barotropic equation of state $p_m = w_m\rho_m$, we get the density of the fluid as $\rho_m = \rho_{m0}a^{-3(1+w_m)}e^{\frac{\alpha(1+3w_m)\sigma}{2}}$.

In next three sections we analyze the effect of dilaton dark energy in three modified gravity models mentioned above.

III. DILATON DARK ENERGY MODEL IN $F(R)$ GRAVITY

In the four-dimensional flat space-time, the action of $f(R)$ gravity with matter can be written as [5],

$$I = \int d^4x \sqrt{-g} \left[\frac{f(R)}{2\kappa^2} + \mathcal{L}_m \right] \quad (8)$$

where the usual Einstein-Hilbert action is generalized by replacing R with $f(R)$, which is an analytic function of R and g is the determinant of the metric tensor $g_{\mu\nu}$ and \mathcal{L}_m is the matter Lagrangian. In this flat space-time ($k = 0$) the Ricci scalar is given by $R = 6\dot{H} + 12H^2$.

From the variation of action (8), we obtain the field equations of this modified gravity is given by,

$$f'(R)R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f(R) + g_{\mu\nu}\square f'(R) - \nabla_\mu \nabla_\nu f'(R) = \kappa^2 T_{\mu\nu}^m \quad (9)$$

where $T_{\mu\nu}^m$ are the energy momentum tensor components the matter field and prime denotes differentiation with respect to the scalar curvature R . The gravitational field equations in flat ($k = 0$) space-time take the form

$$H^2 = \frac{\kappa^2}{3f'(R)}(\rho_m + \rho_c) \quad (10)$$

and

$$\dot{H} = -\frac{\kappa^2}{2f'(R)}(\rho_m + p_m + \rho_c + p_c) \quad (11)$$

where ρ_m being the energy density and p_m is the pressure of all ordinary matter. ρ_c and p_c can be regarded as the energy density and pressure generated due to the difference of $f(R)$ gravity and general relativity, given by [4]

$$\rho_c = \frac{1}{\kappa^2} \left[\frac{1}{2} (-f(R) + Rf'(R)) - 3H\dot{R}f''(R) \right] \quad (12)$$

and

$$p_c = \frac{1}{\kappa^2} \left[\frac{1}{2} (f(R) - Rf'(R)) + (2H\dot{R} + \ddot{R})f''(R) + \dot{R}^2 f'''(R) \right] \quad (13)$$

Now from (2), (3), (6), (7), (10) and (11) we get,

$$\rho_m + \rho_c = f'(R) \left(\rho_m + \frac{1}{2}\dot{\sigma}^2 + V(\sigma) \right) \quad (14)$$

and

$$\rho_m + p_m + \rho_c + p_c = f'(R) (\rho_m + p_m + \dot{\sigma}^2) \quad (15)$$

Now consider the model where $f(R)$ is defined as,

$$f(R) = R + \xi R^\mu + \eta R^{-\nu} \quad (16)$$

Taking the higher derivatives of the above equation w.r.t. R we get,

$$f'(R) = 1 + \xi\mu R^{\mu-1} - \eta\nu R^{-\nu-1}$$

$$f''(R) = \xi\mu(\mu-1)R^{\mu-2} + \eta\nu(\nu+1)R^{-\nu-2}$$

and

$$f'''(R) = \xi\mu(\mu-1)(\mu-2)R^{\mu-3} - \eta\nu(\nu+1)(\nu+2)R^{-\nu-3}$$

Now we take the Hubble parameter H of the form, $H = a_0 a^n$ where $n > 0$. Thus we have, $\dot{H} = na_0^2 a^{2n}$.

Now using the equations (12) - (16) we get,

$$V(\sigma) = -(2a_0^{2n}a_0^2(\eta(1+\nu)(6+n(3+\nu(-5+n+2n\nu))) - 6^{\mu+\nu}(-a_0^{2n}a_0^2(2+n))^{\mu+\nu}(-1+\mu)(6+n(3+\mu(5+n$$

$$\begin{aligned}
& (-1 + 2\mu)))\xi) + e^{\frac{\alpha\sigma}{2}} \kappa^2 (\eta\nu - \nu(2^{1+\mu+\nu} 3^{\mu+\nu} - 6^{\mu+\nu}) (-a^{2n} a_0^2 (2+n))^{\mu+\nu} \mu\xi) \rho_0) \\
& / (2\kappa^2 (6a^{2n} a_0^2 (-a^{2n} a_0^2 (2+n))^\nu (2^{1+\nu} 3^\nu + 6^\nu n) + \eta\nu - 6^{\mu+\nu} (-a^{2n} a_0^2 (2+n))^{\mu+\nu} \mu\xi))
\end{aligned} \tag{17}$$

and

$$\begin{aligned}
\sigma = \int \sqrt{\frac{1}{\kappa^2} (2a^{2n} a_0^2 n (\eta\nu(1+\nu)(1+n+2n\nu) - 6^{\mu+\nu} (-a^{2n} a_0^2 (2+n))^{\mu\nu} (-1+\mu)\mu(-1+n(-1+2\mu))\xi) - a^{-3} e^{\frac{\alpha\sigma}{2}} \kappa^2} \\
\frac{(\eta\nu - 6^{\mu+\nu} (-a^{2n} a_0^2 (2+n))^{\mu+\nu} \mu\xi) \rho_0)}{6a^{2n} a_0^2 (-a^{2n} a_0^2 (2+n))^\nu (2^{1+\nu} 3^\nu + 6^\nu n) + \eta\nu - 6^{\mu+\nu} (-a^{2n} a_0^2 (2+n))^{\mu+\nu} \mu\xi} dt
\end{aligned} \tag{18}$$

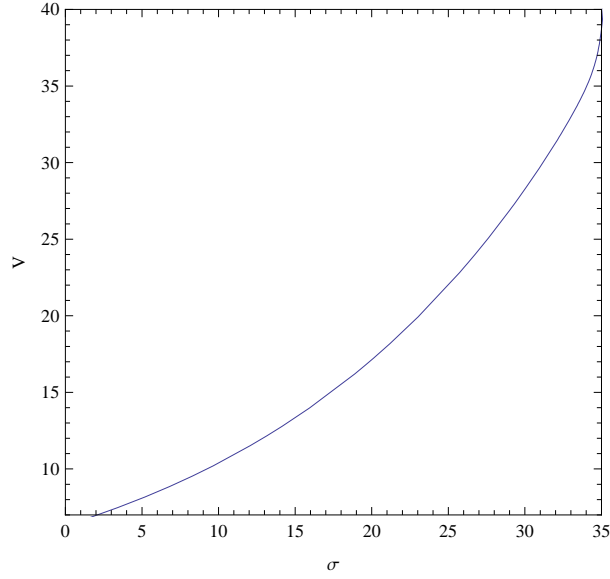


Fig.1

Fig.1 shows the variation of V against σ for $a_0 = 1, n = 1.5$.

From above, we have seen that the expressions of σ and V are very complicated. So V cannot be expressed explicitly in terms of σ . Fig.1 shows the the variation of V against σ for $a_0 = 1, n = 1.5$ in $f(R)$ gravity model. From the figure, we have seen that the potential V always increases with the dilaton field σ .

IV. DILATON DARK ENERGY MODEL IN $F(T)$ GRAVITY

The action for the $f(T)$ gravity is given by [19],

$$S = \frac{1}{2\kappa^2} \int dx^4 [\sqrt{-g} f(T) + \mathcal{L}_m] \tag{19}$$

where T is the torsion scalar, $f(T)$ is general differentiable function of the torsion. Here the torsion scalar T is defined as [9],

$$T = S_\rho^{\mu\nu} T_{\mu\nu}^\rho \tag{20}$$

where,

$$S_\rho^{\mu\nu} = \frac{1}{2} (\delta_\rho^\mu T^{\theta\nu}_\theta - \delta_\rho^\nu T^{\theta\mu}_\theta) - \frac{1}{4} (T^{\mu\nu}_\rho - T^{\nu\mu}_\rho - T_\rho^{\mu\nu}) \tag{21}$$

and

$$T_{\mu\nu}^{\lambda} = e_i^{\lambda}(\partial_{\mu}e_{\nu}^i - \partial_{\nu}e_{\mu}^i) \quad (22)$$

where $e = \sqrt{-g}$. We now assume a flat ($k = 0$) homogeneous and isotropic FRW universe and for this model,

$$e_{\mu}^i = \text{diag}(1, a(t), a(t), a(t)) \quad \text{and} \quad T = -6H^2 \quad (23)$$

The modified Fridmann equations can be written as,

$$12H^2 f'(T) + f(T) = 2\kappa^2 \rho_m \quad (24)$$

and

$$48H^2 \dot{H} f''(T) - (12H^2 + 4\dot{H})f'(T) - f(T) = 2\kappa^2(\rho_m + p_m) \quad (25)$$

where prime denotes the derivatives w.r.t. T .

Now putting $T = -6H^2$, the above set of equations becomes,

$$-2T f'(T) + f(T) = 2\kappa^2 \rho_m \quad (26)$$

and

$$-8T \dot{H} f''(T) + (2T - 4\dot{H})f'(T) - f(T) = 2\kappa^2(\rho_m + p_m) \quad (27)$$

As the equation (26) and (27) will become usual Einstein's equations for a special case when $f(T) = T$ so the equations can be rewritten as

$$H^2 = \frac{\kappa^2}{3}(\rho_m + \rho_T) \quad (28)$$

and

$$\dot{H} = -\frac{\kappa^2}{2}(\rho_m + p_m + \rho_T + p_T) \quad (29)$$

where ρ_T and p_T be the torsion contributions to the energy density and pressure in $f(T)$ gravity.

From the above equations we get,

$$\rho_T = \frac{1}{2\kappa^2}(2T f'(T) - f(T) + 6H^2), \quad p_T = -\frac{1}{2\kappa^2}(-8\dot{H}T f''(T) + (2T - 4\dot{H})f'(T) - f(T) + 4\dot{H} + 6H^2) \quad (30)$$

Now consider a particular modified $f(T)$ gravity model where $f(T)$ is defined as,

$$f(T) = \beta T + \gamma T^m \quad (31)$$

Taking the higher derivatives of the above equation w.r.t. T we get,

$$f'(T) = \beta + m\gamma T^{m-1}$$

$$f''(T) = m(m-1)\gamma T^{m-2}$$

and

$$f'''(T) = m(m-1)(m-2)\gamma T^{m-3}$$

Now we take H of the form, $H = a_0 a^{-n}$ where a_0 is a constant and $n > 0$.

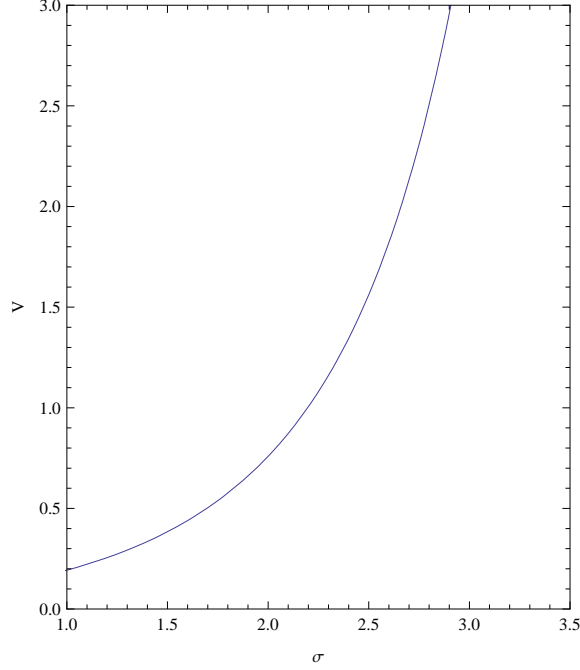


Fig.2

Fig.2 represents the variation of V against σ for $a_0 = 1, n = 1.5$.

Now from (2), (3), (30), (31) and using the above derivatives we get,

$$V(\sigma) = \frac{a^{-2n}}{6a_0^2\kappa^2} (6a_0^4(3 + (-3 + n)\beta) - 6^m a^{4n} (-a^{-2n} a_0^2)^m m \gamma + a^{2n} a_0^2 (6\beta - 6^m (-a^{2n} a_0^2)^m (-1 + 2m)(-3 + mn)\gamma)) \quad (32)$$

and

$$\sigma = \int \sqrt{\frac{(-6 - 6a^{-2n} a_0^2 n)\beta + \frac{1}{a_0^2} (-a^{-2n} a_0^2)^m m (6^m a^{2n} - a_0^2 (6^m - 2^{1+m} 3^m m) n) \gamma}{3\kappa^2}} dt \quad (33)$$

From above, we have seen that the expressions of σ and V are very complicated. So V cannot be expressed explicitly in terms of σ . Fig.2 represents the variation of V against σ for $a_0 = 1, n = 1.5$ in $f(T)$ gravity model. From the figure, we have seen that V always increases with σ .

V. DILATON DARK ENERGY MODEL IN HOŘAVA-LIFSHITZ GRAVITY

In the (3+1) dimensional Arnowitt-Deser-Misner formalism the full metric is written as [39],

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \quad (34)$$

Under the detailed balance condition the full action condition of Hořava-Lifshitz gravity is given by,

$$S = \int dt d^3x \sqrt{g} N \left[\frac{2}{\kappa_1^2} (K_{ij} K^{ij} - \lambda K^2) + \frac{\kappa_1^2}{2\omega^4} C_{ij} C^{ij} - \frac{\kappa_1^2 \mu \epsilon^{ijk}}{2\omega^2 \sqrt{g}} R_{il} \nabla_j R_k^l \right. \\ \left. + \frac{\kappa_1^2 \mu^2}{8} R_{ij} R^{ij} + \frac{\kappa_1^2 \mu^2}{8(3\lambda - 1)} \left(\frac{1 - 4\lambda}{4} R^2 + \Lambda R - 3\Lambda^2 \right) \right] \quad (35)$$

where

$$K_{ij} = \frac{1}{2N}(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i) \quad (36)$$

is the extrinsic curvature and

$$C^{ij} = \frac{\epsilon^{ikl}}{\sqrt{g}} \nabla_k (R_i^j - \frac{1}{4} R \delta_l^j) \quad (37)$$

is known as Cotton tensor and the covariant derivatives are defined with respect to the spatial metric g_{ij} . ϵ^{ijk} is the totally antisymmetric unit tensor, λ is a dimensionless coupling constant and the variable κ_1 , ω and μ are constants with mass dimensions -1 , 0 , 1 respectively.

Now, in order to focus on cosmological frameworks, we impose the so called projectability condition and use a FRW metric we get,

$$N = 1, g_{ij} = a^2(t) \gamma_{ij}, N^i = 0 \quad (38)$$

with

$$\gamma_{ij} dx^i dx^j = \frac{dr^2}{1 - kr^2} + r^2 d\Omega_2^2, \quad (39)$$

where $k = 0, -1, +1$ corresponding to flat, open and closed respectively. By varying N and g_{ij} , we obtain the non-vanishing equations of motions:

$$H^2 = \frac{\kappa_1^2}{6(3\lambda - 1)} \rho_m + \frac{\kappa_1^2}{6(3\lambda - 1)} \left[\frac{3\kappa^2 \mu^2 k^2}{8(3\lambda - 1)a^4} + \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \right] - \frac{\kappa^4 \mu^2 k \Lambda}{8(3\lambda - 1)^2 a^2} \quad (40)$$

and

$$\dot{H} + \frac{3}{2} H^2 = -\frac{\kappa_1^2}{4(3\lambda - 1)} p_m - \frac{\kappa_1^2}{4(3\lambda - 1)} \left[\frac{\kappa^2 \mu^2 k^2}{8(3\lambda - 1)a^4} - \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \right] - \frac{\kappa^4 \mu^2 k \Lambda}{16(3\lambda - 1)^2 a^2} \quad (41)$$

The term proportional to a^{-4} is the usual “dark radiation”, present in Hořava-Lifshitz cosmology while the constant term is just the explicit cosmological constant. For $k = 0$, there is no contribution from the higher order derivative terms in the action. However for $k \neq 0$, their higher derivative terms are significant for small volume i.e., for small a and become insignificant for large a , where it agrees with general relativity. As a last step, requiring these expressions to coincide the standard Friedmann equations, in units where $c = 1$,

$$G_c = \frac{\kappa_1^2}{16\pi(3\lambda - 1)} \quad (42)$$

$$\frac{\kappa_1^4 \mu^2 \Lambda}{8(3\lambda - 1)^2} = 1 \quad (43)$$

where G_c is the “cosmological” Newton’s constant. We mention that in theories with Lorentz invariance breaking (such is Hořava-Lifshitz one) the “gravitational” Newton’s constant G , that is the one that is present in the gravitational action, does not coincide with G_c , that is the one that is present in Friedmann equations, where

$$G = \frac{\kappa_1^2}{32\pi} \quad (44)$$

as it can be straightforwardly read from the action. In the IR ($\lambda = 1$) where Lorentz invariance is restored, $G_c = G$. Using the above identifications, we can re-write the Friedmann equations as,

$$H^2 + \frac{k}{a^2} = \frac{l^2}{3}\rho_m + \frac{k^2}{2\Lambda a^4} + \frac{\Lambda}{2} \quad (45)$$

and

$$\dot{H} - \frac{k}{a^2} = -\frac{l^2}{2}(\rho_m + p_m) - \frac{k^2}{\Lambda a^4} \quad (46)$$

where $l^2 = 8\pi G_c$. Again the usual Einstein's field equations are

$$H^2 + \frac{k}{a^2} = \frac{\kappa^2}{3}(\rho_m + \rho_h) \quad (47)$$

and

$$\dot{H} - \frac{k}{a^2} = -\frac{\kappa^2}{2}(p_m + p_h + \rho_m + \rho_h) \quad (48)$$

Here, ρ_h and p_h are the contribution of energy density and pressure by Hořava-Lifshitz gravity. Now from (45) and (47) we get,

$$\rho_h = \left(\frac{l^2}{\kappa^2} - 1\right)\rho_m + \frac{3}{2\kappa^2}\left(\frac{k^2}{\Lambda a^4} + \Lambda\right) \quad (49)$$

Equating (46) and (48) we get,

$$p_h + \rho_h = \left(\frac{l^2}{\kappa^2} - 1\right)(\rho_m + p_m) + \frac{k^2}{\kappa^2 \Lambda a^4} \quad (50)$$

Thus $\rho_\sigma = \rho_h$ and $p_\sigma = p_h$ give,

$$\sigma = \int \sqrt{\left(\frac{l^2}{\kappa^2} - 1\right)(\rho_m + p_m) + \frac{k^2}{\kappa^2 \Lambda a^4}} dt \quad (51)$$

and

$$V(\sigma) = \frac{1}{2}\left(\frac{l^2}{\kappa^2} - 1\right)(\rho_m - p_m) + \frac{k^2}{\kappa^2 \Lambda a^4} + \frac{3\Lambda}{2\kappa^2} \quad (52)$$

Fig.3 represents the variation of V against σ for $\rho_0 = 1, k = 1$ in Hořava-Lifshitz gravity model. From the figure, we have seen that V always increases with σ .

VI. DISCUSSIONS

Our aim in this work is to construct a cosmological model by using modified gravity theories of some particular forms as the contribution of a effective dark energy. Also an additional dark fluid has been included to play an important cosmological role. In fact we consider two accelerated epoch of the universe described by a model where the mysterious energy generated from two separate source. So in our model, the acceleration of the universe become faster than the normal dark energy effect or modified gravity effect separately and give rise to an super acceleration phase. In this work, we have considered dilaton dark energy model in Weyl-scaled induced gravitational theory in presence of barotropic fluid. It is to be noted that the dilaton field behaves as a quintessence. Here we have discussed the role of dilaton dark energy in modified gravity theories namely, $f(R)$, $f(T)$ and Hořava-Lifshitz gravities and graphically analyzed the behaviour of the dilaton field and the corresponding potential in respect to these modified gravity theories instead of Einstein's gravity. In $f(R)$

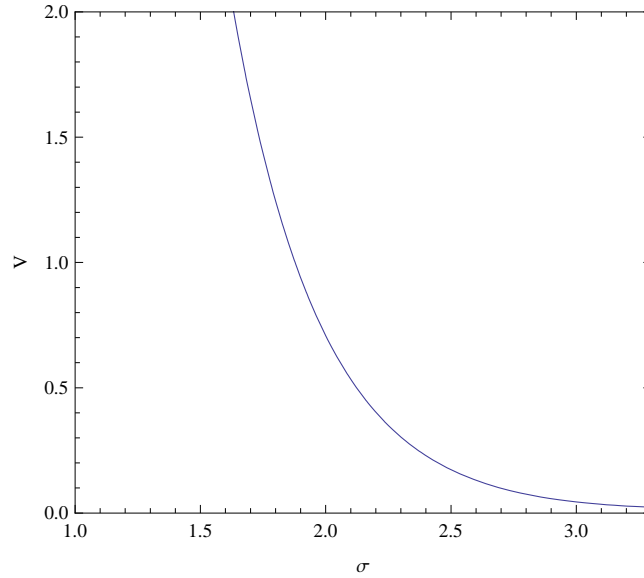


Fig.3

Fig.3 represents the variation of V against σ for $\rho_0 = 1, k = 1$.

and $f(T)$ gravities, we have considered some particular forms of $f(R)$ and $f(T)$ and we have shown that the potentials always increase with the dilaton fields. But in Hořava-Lifshitz gravity, it has been seen that the potential always decreases as dilation field increases.

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